advance towards the more refrangible parts of the spectrum in the inverse order of their atomic weights.* The correlation of the spectral reactions of thallium, indium, and gallium with the other properties of these elements is of further interest from the fact that their arc spectra (below the ultra-violet) are represented by homologous pairs of lines in the order of their atomic weights. Tl = 204; λ6560, 5349. In = 113·4; λ4510, 4101. Ga = 70; λ4170, 4031. The intervals of space between each homologous pair of lines, as will be seen, increase in the same order. These relations are further represented in the subjoined diagram, reduced from the scale of Ångström's normal spectrum.

It would be interesting to know if the arc spectrum of scandium is represented by a similar pair of lines in the ultra-violet, as I have already suggested in the paper referred to before this elementary substance was discovered.

III. "The Potential of an Anchor Ring." By F. W. Dyson, M.A., Fellow of Trinity College, Cambridge, Isaac Newton Student in the University of Cambridge. Communicated by Professor J. J. Thomson, F.R.S. Received March 16, 1893.

(Abstract.)

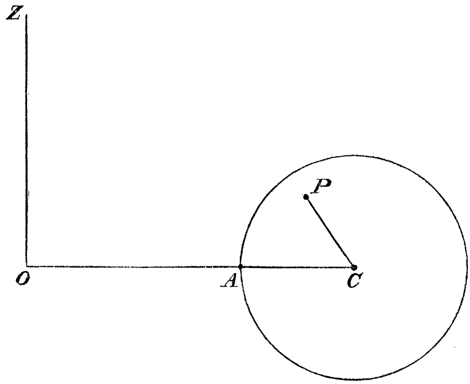
1. This paper is a continuation of some researches on rings published in the 'Phil. Trans.,' 1893. In that paper the potential of an anchor ring was found at all external points; here it is determined for internal points. The annular form of rotating gravitating fluid was considered; the stability of such a ring is investigated here. In addition, the potential of a ring of elliptic cross-section, being of interest in connexion with Saturn, is obtained. Besides this, the similarity of the methods and of the analysis employed has led me to put in this paper also the determination of the steady motion of a single vortex ring of finite cross-section and the motion of several fine vortex rings on the same axis.

2. Let the figure represent a section through the axis Oz of an anchor ring. O is the centre of the ring, C the centre of the cross-section, P any point inside the ring.

$$\text{Let OC} = c, \qquad \text{CA} = a, \qquad \text{CP} = \text{R}, \qquad \angle\,\text{ACP} = \chi.$$

Then it is shown that

* 'Proceedings and Memoirs of the Manchester Lit. and Phil. Society,' 1878—1886.



$$\frac{R^n}{a^n}\cos n\chi+\frac{a}{2c}\cdot\frac{R^{n+1}}{a^{n+1}}\cos\overline{n-1}\chi+\left(\frac{a}{2c}\right)^2\cdot\frac{R^{n+2}}{a^{n+2}}\left\{\tfrac{1}{2}\frac{2n+1}{2n+2}\cos n\chi+\frac{1.3}{2.4}\right.$$

$$\left.\cos\overline{n-2}\chi\right\}+\ldots\ldots+\left(\frac{a}{2c}\right)^p\cdot\left(\frac{R}{a}\right)^{n+p}\left\{\tfrac{1}{2}\frac{(2n+1)\ldots\ldots(2n+2p-3)}{(2n+2)\ldots\ldots(2n+2p-2)}\right.$$

$$\cos(n+p-2)\chi+\frac{p-1}{1}\cdot\frac{1.3}{2.4}\cdot\frac{(2n+1)\ldots\ldots(2n+2p-5)}{(2n+2)\ldots\ldots(2n+2p-4)}\cos(n+p-4)\chi$$

$$+\frac{(p-1)(p-2)}{2\,!}\cdot\frac{1.3.5}{2.4.6}\cdot\frac{(2n+1)\ldots\ldots(2n+2p-7)}{(2n+2)\ldots\ldots(2n+2p-6)}\cos(n+p-6)\chi$$

$$\left.+\ldots\ldots\right\}+\text{\&c.}$$

is a solution of Laplace's equation.

A similar expression holds when for cosines we write sines.

Writing now

$$\sigma=\frac{a}{c}\quad\text{and}\quad L=\log\frac{8c}{a},$$

the potential of a solid anchor ring at internal points is found as far as terms in σ^4. As far as terms in σ^2 it is given by

$$V=2\pi a^2\left\{L+\tfrac{1}{2}\left(1-\frac{R^2}{a^2}\right)+\sigma\left[\frac{L-1}{2}\cdot\frac{R}{a}-\frac{R^3}{8a^3}\right]\cos\chi+\sigma^2\left[-\frac{L-\frac{1}{4}}{16}\right.\right.$$

$$\left.\left.+\frac{L-1}{8}\cdot\frac{R^2}{a^2}-\tfrac{3}{64}\frac{R^4}{a^4}+\left(\frac{3\left(L-\frac{5}{4}\right)}{16}\frac{R^2}{a^2}-\tfrac{5}{96}\cdot\frac{R^4}{a^4}\right)\cos 2\chi\right]\right\}.$$

The work done in collecting the ring from infinity is

$$\frac{M^2}{2\pi c}\left\{L+\tfrac{1}{4}-\frac{L-\frac{2}{3}}{8}\sigma^2-\frac{3\left(L-\frac{19}{12}\right)}{512}\sigma^4-\ldots\ldots\right\}.$$

A distribution of matter on the ring of surface density $\beta_n \cos n\chi$ gives on the ring the potential V where

$$\frac{V}{2\pi a\beta_n} = \frac{\cos n\chi}{n} + \frac{\sigma}{4n}\left\{\frac{\cos(n+1)\chi}{n+1} - \frac{\cos(n-1)\chi}{n-1}\right\}$$

$$+\frac{\sigma^2}{16n}\left\{\frac{2n+3}{(n+1)(n+2)}\cos(n+2)\chi - \frac{2n-3}{(n-1)(n-2)}(\cos n-2)\chi\right\}$$

$$+\text{\&c.}$$

3. The stability of an annulus of rotating fluid is considered for three kinds of disturbances: *fluted*, in which case the ring remains symmetrical about its axis, but its cross-section ceases to be circular; *twisted*, in which case the central circle of the ring is deformed, but the cross-section is undisturbed; *beaded*, in which case the central line remains circular, and the cross-section is a circle, but one of varying radius. It is proved that when the cross-section is small compared with the radius the annular form is stable for fluted and twisted disturbances, but is broken up by beaded waves.

4. In Laplace's proof that Saturn's rings cannot be continuous fluid rotating in relative equilibrium, he assumes that the attraction of the ring at a point on the surface is the same as that of a cylinder. Madame Kowalewski, in her memoir, uses a method which applies only to rings of nearly circular cross-section. Here I have found the potential of a ring of elliptic cross-section. The results are complicated. For a very flat ring of mass M of mean radius c, and whose cross-section has a semi-major axis a, the exhaustion of potential energy is if $(a/c)^2$ is neglected

$$\frac{M^2}{2\pi c}\cdot\left(\log\frac{16c}{a}+\tfrac{1}{4}\right).$$

As applied to Saturn, the result obtained is that for the ring to be continuous fluid its density would have to be about 100 times the density of the planet.

5. Let m be the strength and c the mean radius of a vortex ring. Let its cross-section be given by

$$R = a\{1+\beta_2\cos 2\chi+\beta_3\cos 3\chi+\beta_4\cos 4\chi+\ldots\}.$$

Then β_2, β_3, β_4, &c., are of the 2nd, 3rd, 4th,.... orders in a/c. Their values are obtained as far as $(a/c)^4$.

The velocity of the ring is found to be

$$\frac{M}{2\pi c}\left\{\log\frac{8c}{a}-\frac{1}{4}-\frac{12\log\frac{8c}{a}-15}{32}\left(\frac{a}{c}\right)^2\right\}.$$

$$\beta_2 = -\frac{12L-17}{32}\left(\frac{a}{c}\right)^2,$$

&c.

The results agree with those obtained by Mr. Hicks, *viâ* toroidal functions. The fluted oscillations are very briefly discussed.

6. The motion of a number of fine vortex rings on the same axis is discussed. Equations are obtained giving the forward velocity and the rate of increase of the radius for each ring. Let m_1 be the strength, c_1 the mean radius, a_1 the radius of the cross-section, z_1 the distance of the centre of the ring along the axis of z.

It is shown that the kinetic energy of the system is given by

$$T = 8\Sigma\left\{\frac{m_1^2c_1}{2}\left(\log\frac{8c_1}{a_1}-\frac{7}{4}\right)\right.$$
$$\left.+m_1m_2\int_0^\pi\frac{c_1c_2\cos\phi\,d\phi}{\sqrt{\{(z_2-z_1)^2+c_1^2-2c_1c_2\cos\phi+c_2^2\}}}\right\}.$$

The equations of motion are

$$m_1c_1\dot{z}_1 = \frac{1}{8\pi}\cdot\frac{\partial T}{\partial c}. \qquad -m_1c_1\dot{c}_1 = \frac{1}{8\pi}\cdot\frac{\partial T}{\partial z_1}.$$

The integral expressing the constancy of momentum takes the simple form

$$\Sigma(m_1c_1^2) = \text{const.}$$

The following particular cases are worked out in detail:—

1. The motion of a ring following another of equal strength.
2. The direct approach of a ring to a fixed plane.
3. The motion of a ring over a spherical obstacle.

IV. "Analogy of Sound and Colour.—Comparison of the Seven Colours of the Rainbow with the Seven Notes of the Musical Scale, as determined by the Monochord, and of the Wave-lengths of Colour and Sound." By J. D. MACDONALD, M.D., F.R.S. Received March 13, 1893.